HYDRAULICS

Dr. Basim Shabaa Abd

Dr. Fadhil Mohammed Dahir

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Dr. Basim Shabaa Abd

Assistant Professor of Water Resources Engineering

Middle Technical University

Dr. Fadhil Mohammed Dahir

Assistant Professor of Water Resources Engineering

AL-Furat AL-Awast Technical University

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Preface

This book was written as a guidebook on fluid mechanics for students or junior engineers studying water resources or civil engineering. The main purpose of this book is to present and provide the basic principles of fluid mechanics and illustrate their applications in various fields of engineering.

In this book, effort has been made to introduce students to hydraulics by making explanations easy to understand, including recent information and comparing the theories with actual phenomena.

This book consists of eleven chapters. First chapter includes a historical perspective and general definitions on the subject of hydraulics. The second chapter addresses three cases of substance in addition to the study of the physical properties of fluids. The third and fourth chapters included a study of hydrostatic pressure on surfaces. Fluid flow has been studied in this book through the fifth and sixth chapters with illustrative examples. The seventh and eightieth chapters included a study of fluid measurements and momentum equation respectively. Chapter nine introduce a definition of open channel flow is flow of liquid in a conduit and study an open channel hydraulics. Pumps in hydraulics applications have been studied in chapter ten. The purpose of chapter eleven is to introduce students to some ready-to-computer applications as well as to give an introduction to how to use Visual Basic Studio 2008 language to solve different cases of pipeline problems.

All of the examples in this text use a consistent problem-solving methodology. Each example highlights the key elements of analysis: Given, Find, Solution, and Comment.

Authors
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CHAPTER EIGHT

IMPLUSE AND MOMETUM EQUATION

8.1 Introduction

In many engineering applications, it requires to find the forces that affect the solids bodies resulting from the flow of fluids.A momentum is a useful tool for the solution of fluid flow problems.Momentum of a body may be defined as the product of its mass and velocity. Since velocity is a vector quantity, momentum is a vector quantity. There are many typical applications for momentum principle such as calculating forces for jets, nozzles and pipe sections.

Momentum equation will be derived from Newton's second law.

Newton's second law of motion for a system is

$$\sum F = m.a \qquad \text{...............} \ \mathbf{(8.1)}$$

where $\sum F$ is sum of external forces acting on the system, (m) is the mass, (a) is acceleration effected in the direction of the force (F), and ($m.a$) is time rate of change of the momentum of the system. The momentum can be expressed as

$$M = m.V \qquad \text{...............} \ \mathbf{(8.2)}$$

where

M = momentum, (MLT^{-1}),

m = mass (M), and

V = velocity, (LT^{-1}).

The momentum of a body remains the same as long as there is no external force acting on it. Linear momentum equation for fluids can be developed using Newton's second Law which states that sum of all forces must equal the time rate of change of the momentum,

$$\sum F = \frac{d(m.V)}{dt} \qquad \text{.............} \ \mathbf{(8.3)}$$

8.2 Steady Flow Momentum Equation

In fluid mechanics the analysis of motion is performed in the same way as in solid mechanics - by use of Newton's laws of motion. Account is also taken for the special properties of fluids when in motion.

The momentum equation is a statement of Newton's second Law and relates the sum of the forces acting on an element of fluid to its acceleration or rate of change of momentum. We will probably recognize the equation *F* = *ma* which is used in the analysis of solid mechanics to relate applied force to acceleration.

Newton's second law can be written as below:

The Rate of change of momentum of a body is equal to the resultant force acting on the body, and takes place in the direction of the force.

Let us consider the equilibrium of the streamtube as in Fig. 8.1.The coordinates direction are x and y. We start by assuming that we have *steady* flow which is *non-uniform* flowing in a stream tube.

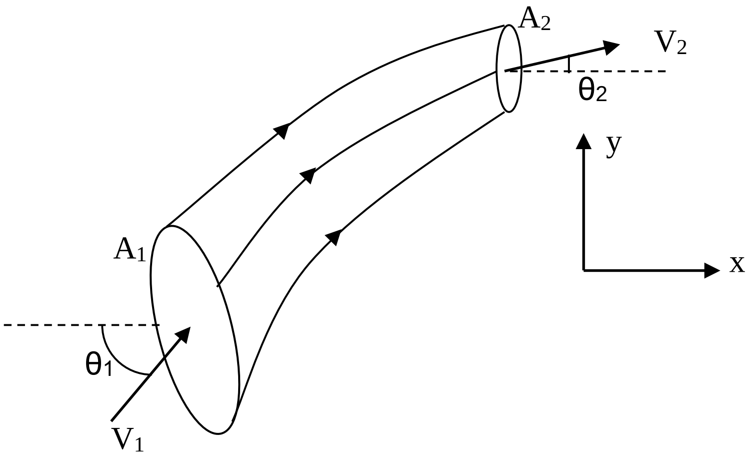


Figure. 8.1 A streamtube in two-dimensions.

In time δt a :, so the volume entering the streamtube in the time δt is as below:

Volume entering the streamtube= (area). (distance), thus,

$$\delta Vo = A_1.V_1.\delta t \quad \text{............. (8.4)}$$

where δVo is a volume entering the streamtube. This has mass:

Mass entering the streamtube= (volume). (density ρ)

$$\delta Mass = \rho_1.A_1.V_1.\delta t \quad \text{............. (8.5)}$$

and momentum is:

Momentum of fluid entering the streamtube= (mass). (velocity)

Momentum of fluid entering the streamtube = $M = \rho_1.A_1.V_1.\delta t.V_1 = \rho_1.A_1.V_1^2.\delta t$

Similarly, at the exit, we can obtain an expression for the momentum leaving the streamtube:

Momentum of fluid leaving the streamtube = $\rho_2.A_2.V_2.\delta t.V_2 = \rho_2.A_2.V_2^2.\delta t$

We can now calculate the force exerted by the fluid using Newton's second law. The force F is equal to the rate of change of momentum. So

Force = rate of change of momentum

$$F = \frac{(\rho_2.A_2.V_2^2.\delta t - \rho_1.A_1.V_1^2.\delta t)}{\delta t} \qquad \text{.............. } \mathbf{(8.6)}$$

We know from continuity that

$$Q = A_1.V_1 = A_2.V_2$$

where Q is a discharge for the flow system.

If we have a fluid of constant density, i.e. $\rho = \rho_1 = \rho_2$, then we can write

$$F = \rho.Q(V_2 - V_1) \qquad \text{.............. } \mathbf{(8.7)}$$

This force F is acting in the direction of the flow of the fluid.

At the inlet the velocity vector, V_1, makes an angle, θ_1, with the x-axis, while at the outlet V_2 make an angle θ_2. In this case we consider the forces by resolving in the directions of the co-ordinate axes. Then for Fig. 8.1, assuming uniform velocity distribution at the two relevant sections, the forces in x and y directions are as below:

The force in the x-direction

F_x=Rate of change of momentum in x-direction

$$F_x = \rho.Q(V_2.\cos\theta_2 - V_1.\cos\theta_1)$$

$$F_x = \rho.Q((V_2)_x - (V_1)_x) \qquad \text{.............. } \mathbf{(8.8)}$$

The force in the y-direction

F_y=Rate of change of momentum in y-direction

$$F_y = \rho.Q(V_2.\sin\theta_2 - V_1.\sin\theta_1)$$

$$F_y = \rho.Q((V_2)_y - (V_1)_y) \qquad \text{.............. } \mathbf{(8.9)}$$

The resultant force

The resultant force F as illustrated in Fig. 8.2 is calculated based on F_x and F_y , as in the following equation:

$$F = \sqrt{F_x^2 + F_y^2} \qquad \text{.............. } \mathbf{(8.10)}$$

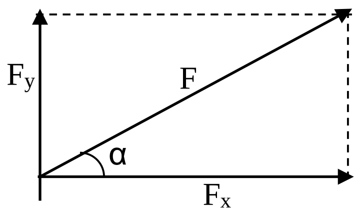


Figure 8.2 A resultant force and its direction

The angle which the resultant force acts α is given by:

$$\alpha = \tan^{-1}(\frac{F_y}{F_x}) \qquad \text{………….. } \mathbf{(8.11)}$$

8.3 Momentum Equation for Change in Velocity

There are many hydraulic applications for fluid momentum which is produced as a result of change in the values of flow velocities. A nozzle attached to a pipe line as shown in Fig. 8.3 provides a good example of a rapid change in velocity.

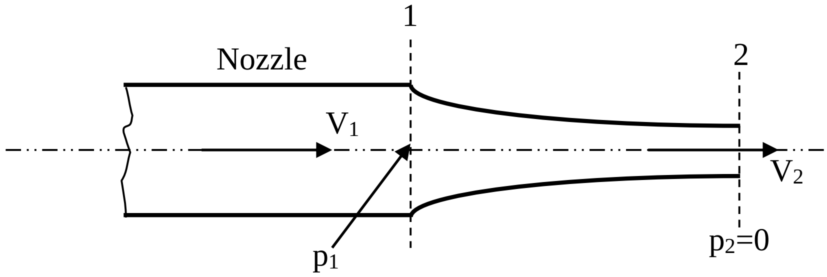


Figure 8.3 A nozzle in a pipeline.

Fig.8.4 shows the forces acting on the nozzle for the part between sections 1 and 2. Application of the momentum equation between sections 1 and 2 will yield a direct solution. The component forces are the hydrostatic forces p_1A_1 and p_1A_1 and the force F_x exerted by the nozzle on the fluid.

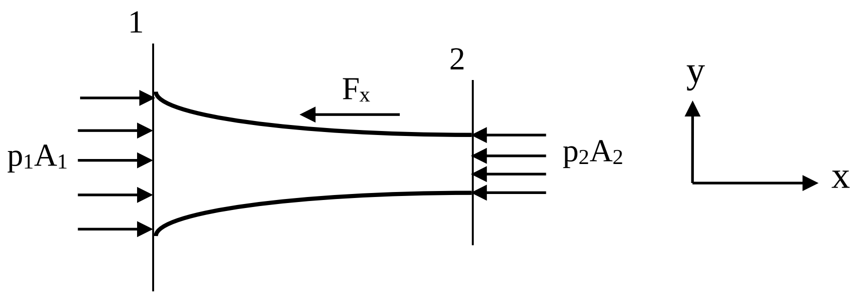


Figure 8.4 Force acting on the nozzle.

Rate of change of momentum is $\rho.Q(V_2 - V_1)$, therefore

$$p_1A_1 - p_2A_2 - F_x = \rho.Q(V_2 - V_1)$$

p_2 is atmospheric pressure=0, then

$$F_x = p_1A_1 - \rho.Q(V_2 - V_1) \qquad \text{………….. } \mathbf{(8.12)}$$

8.4 Momentum Equation for Change in Velocity and Direction

Consider a reducing bend with deviation in vertical plane is shown in Fig.8.5.

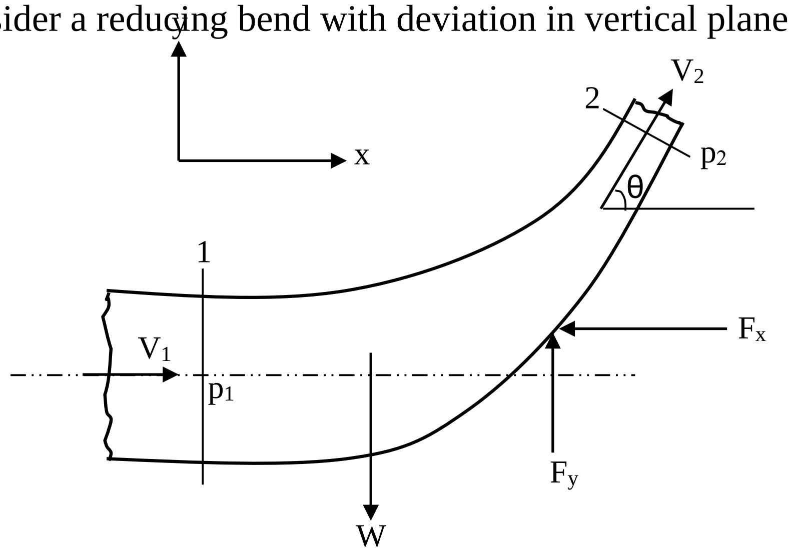


Figure 8.5 A nozzle in a pipeline.

Application of momentum equation neglecting any energy losses, we obtain,

For x direction

$$p_1 A_1 - p_2 A_2 \cos\theta - F_x = \rho Q(V_2 \cos\theta - V_1)$$

$$F_x = p_1 A_1 - p_2 A_2 \cos\theta - \rho Q(V_2 \cos\theta - V_1) \qquad \text{………….. } \mathbf{(8.13)}$$

For y direction

$$F_y - W - p_2 A_2 \sin\theta = \rho Q(V_2 \sin\theta - 0)$$

$$F_y = p_2 A_2 \sin\theta + W + \rho Q(V_2 \sin\theta - 0) \qquad \text{………….. } \mathbf{(8.14)}$$

where W is the weight of fluid between the two sections 1 and 2.

F_x and F_y are the resultant F which is the force exerted by the bend on fluid. The resultant equation is

$$F = \sqrt{F_x^2 + F_y^2}$$

8.5 Momentum of Free Jet

A change of momentum occurs when a free jet of a fluid is deflected by a solid surface. Below equations that representing the impact of changing the direction of free jet for two types of surfaces lie in a horizontal plane.

1. **Straight horizontal surface:**

Consider a straight horizontal plate exposed to a free jet as shown in Fig. 8.6. The pressure in all sections 1, 2, and 3 is atmospheric that equal to zero. Application of momentum equation, we obtain,

<u>In x-direction</u>

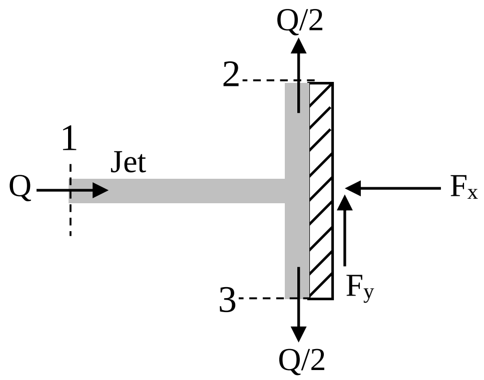


Figure 8.6 Free jet on straight surface.

$$\sum F_x = \rho.Q(\Delta V)_x$$

$$-F_x = (\rho.Q.V)_{out} - (\rho QV)_{in}$$

$$-F_x = (\rho.Q.(V_x)_2 + \rho Q(V_x)_3 - \rho QV_1$$

$(V_x)_2 = (V_x)_3 = 0$, then

$$F_x = \rho.Q.V_1 \quad \text{…………….. } \mathbf{(8.15)}$$

<u>In y-direction</u>

$$\sum F_y = \rho.Q(\Delta V)_y$$

$$F_y = (\rho.Q.V)_{out} - (\rho QV)_{in}$$

$$V_2 = V_3$$

$$F_y = (\rho.\frac{Q}{2}V_2 + \rho\frac{Q}{2}V_3) - \rho Qx0)$$

$$F_y = (\rho.\frac{Q}{2}V_2 + \rho\frac{Q}{2}(-V_2) - \rho Qx0)$$

$$F_y = 0$$

Then, the resultant force is:

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{F_x^2 + 0}$$

$$F = F_x \quad \text{…………….. } \mathbf{(8.16)}$$

2. **Horizontal curved surface at an angle of 90°:**

As shown in Fig. 8.7, a horizontal jet of water strikes a curved surface, and is turned through an angle 90°. Determine the anchoring force needed to hold the surface stationary.

In x-direction

$$\sum F_x = \rho.Q(\Delta V)_x$$

$$-F_x = (\rho.Q.V)_{out} - (\rho QV)_{in}$$

$V_1=V_2=V$, then

$$-F_x = \rho.Q \times 0 - \rho QV_1$$

$$F_x = \rho.Q.V_1 = \rho QV \qquad \text{………….. } \mathbf{(8.17)}$$

In y-direction

$$\sum F_y = \rho.Q(\Delta V)_y$$

$$F_y = (\rho.Q.V)_{out} - (\rho QV)_{in}$$

$$F_y = \rho.Q(V_2 - 0)$$

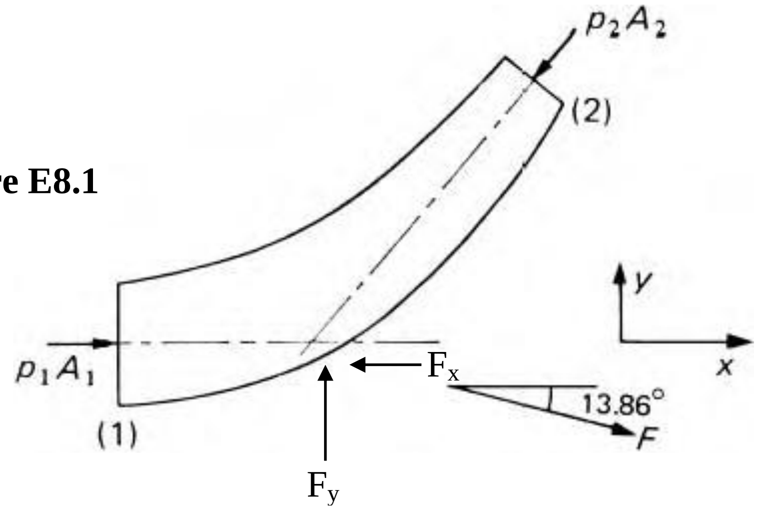
$$F_y = \rho.Q.V_2 = \rho QV \qquad \text{………….. } \mathbf{(8.18)}$$

Example 8.1

GIVEN A 45° reducing pipe-bend (in a horizontal plane) tapers from 600 mm diameter at inlet to 300 mm diameter at outlet as shown in Fig. E8.1. The gauge pressure at inlet is 140 kPa and the rate of flow of water through the bend is 0.425m^3 /s.

FIND Calculate the net resultant horizontal force exerted by the water on the bend.

Figure E8.1



SOLUTION

Assuming uniform conditions with straight and parallel streamlines at inlet and outlet, we have:

$$V_1 = \frac{0.45}{\frac{\pi}{4}(0.6)^2} = 1.503 m/\sec$$

$$V_2 = \frac{0.45}{\frac{\pi}{4}(0.3)^2} = 6.01 m/\sec$$

By the energy equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$$\frac{1.4x10^5}{9810} + \frac{1.503^2}{2g} + 0 = \frac{p_2}{9810} + \frac{6.01^2}{2g} + 0$$

In the *x* direction, force on water in control volume

$$p_1 A_1 - p_2 A_2 - F_x = \rho Q(V_2 \cos 45 - V_1)$$

where F_x represents *x*-component of force exerted by bend *on* water.

Therefore

$$1.4\times10^5.\frac{\pi}{4}(0.6)^2 - 1.231\times10^5.\frac{\pi}{4}(0.3)^2 - F_x = 1000\times0.425(6.01\cos 45 - 1.503)$$

$$F_x = 32260N$$

In the *y* direction, force on water in control volume

$$-p_2 A_2 \sin 45 + F_y = \rho Q(V_2 \sin 45 - 0)$$

$$-1.231\times10^5.\frac{\pi}{4}(0.3)^2 \sin 45 + F_y = 1000\times0.425(6.01\sin 45 - 0)$$

$$F_y = 7960N$$

Therefore total net force exerted on water is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(32260)^2 + (7960)^2} = 33230N$$

Force *F* exerted on bend is equal and opposite of this.

$\alpha = \tan^{-1}\frac{7960}{32260} = 13.86^o$ to the x-direction

COMMENT

For a pipe-bend not entirely in the horizontal plane the weight of the fluid W contributes to the force causing the momentum change.

Example 8.2

GIVEN A stationary divider blade divides the water free jet of 2.7 m^3/s as shown in Fig. E 8.2.

FIND Calculate the force exerted by the jet on the blade (F_x and F_y) if a jet velocity is 30 m/s.

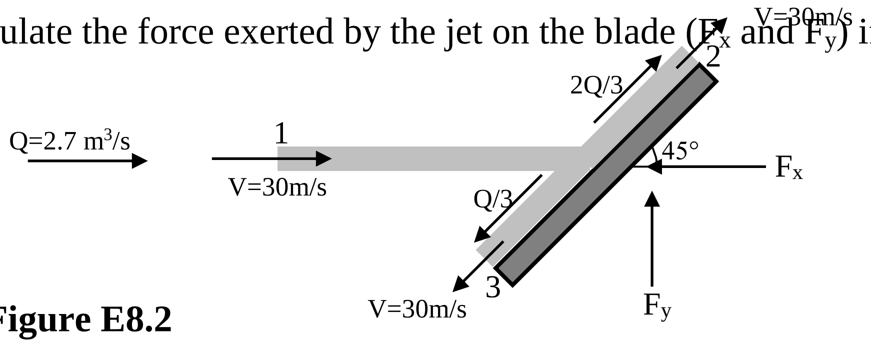


Figure E8.2

SOLUTION

$$\sum F_x = (\rho.QV)_{out} - (\rho QV)_{in}$$

$$-F_x = \rho.Q_2.V_2 \cos 45 + \rho Q_3(-V_3)\cos 45 - \rho Q_1 V_1$$

$$Q_2 = \frac{2Q}{3} = \frac{2\times 2.7}{3} = 1.8 m^3/s$$

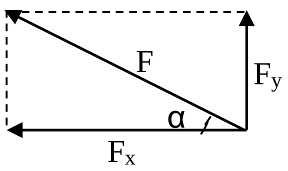
$$Q_3 = \frac{Q}{3} = \frac{2.7}{3} = 0.9 m^3/s$$

$$-F_x = 1000.(1.8\times 30\cos 45 + 0.9\times(-30)\cos 45 - 2.7\times 30)$$

$$F_x = 61908 N \longleftarrow$$

$$\sum F_y = (\rho.QV)_{out} - (\rho QV)_{in}$$

$(V_y)_1 = 0$, then



$$F_y = \rho.Q_2.V_2 \sin 45 + \rho Q_3(-V_3)\sin 45 - \rho Q_1 x0$$

$$F_y = 1000.(1.8\times 30\sin 45 + 0.9\times(-30)\sin 45)$$

$$F_y = 19092 N \uparrow$$

the force exerted by the jet on the blade is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(61908)^2 + (19092)^2} = 64785 N$$

$$F = 64.785 kN$$

$$\alpha = \tan^{-1}(\frac{F_x}{F_y})$$

Force *F* exerted on the blade is

$\alpha = \tan^{-1}\frac{19092}{61908} = 17.1^o$ to the x-direction

COMMENT

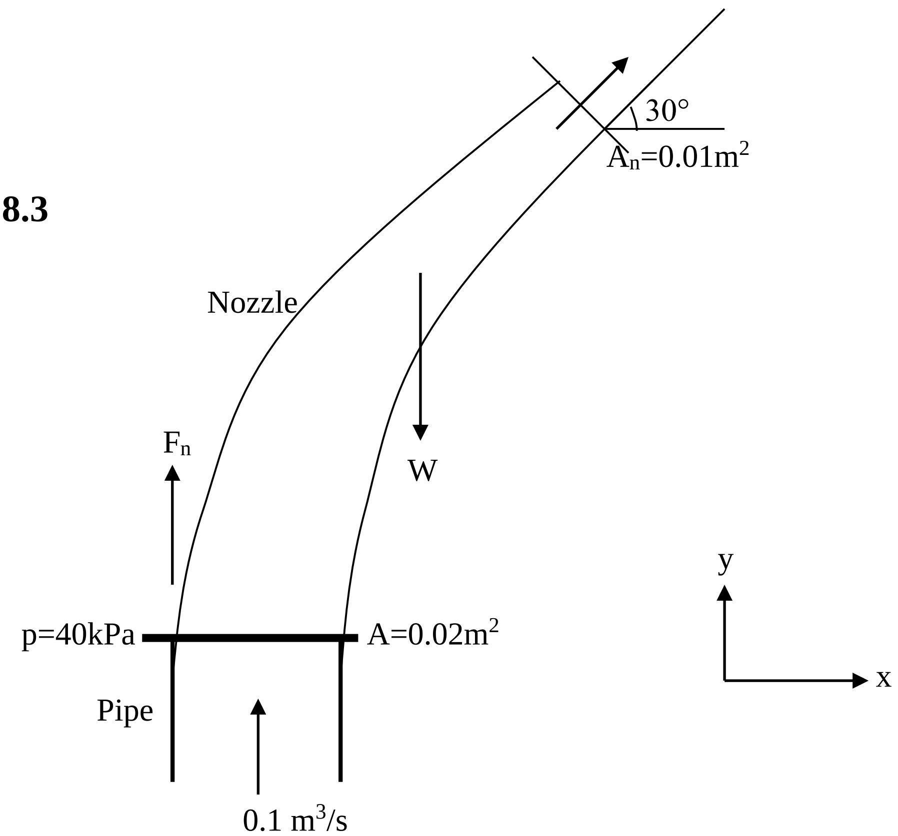
In order to solve any question, that assumption requires specific directions of the forces in both directions x and y.

Example 8.3

GIVEN A nozzle is attached to a vertical pipe and discharges water into the atmosphere as shown in Fig. E8.3.In addition, we have the following information:

Discharge	Q	0.01 m^3/s
Cross-sectional area of the pipe	A	0.02 m^2
Exit area of the nozzle	A_n	0.01 m^2
Pressure at the flange	P	40 kPa
Nozzle weight	W	200 N
Volume of water in the nozzle	Vo	0.012 m^3

Figure E8.3



FIND Determine the vertical component of the anchoring force required to hold the nozzle in place.

SOLUTION

$$V_1 = \frac{Q_1}{A} = \frac{0.1}{0.02} = 5m/s$$

$$V_2 = \frac{Q_1}{A_n} = \frac{0.1}{0.01} = 10m/s$$

Momentum equation in y-direction

$$\sum F_y = (\rho.Q.V)_{out} - (\rho QV)_{in}$$

If F_n is the force on the water that makes in turn, assume it is in the positive y-direction, W_N is nozzle weight. Thus, momentum equation can be written as:

$$p_1 A_1 + F_n - W_{H_2O} - W_N - p_2 A_2 \sin 30 = \rho Q(V_2 \sin 30 - V_1)$$

$$p_2 = 0$$

$$W_{H20} = \gamma_{H_2O} x Volume_{H_2O} = 9800x0.012 = 117.6N$$

$$F_n = -p_1.A1 + W_{H_2O} + W_N + \rho Q(V_2 \sin 30 - V_1)$$

$$F_n = -40000 \times 0.02 + 117.6 + 200 + 1000 \times 0.1(10 \sin 30 - 5)$$

$$F_n = -482N$$

COMMENT

The minus sign implies that the force on the water is in the downward direction, the force from the water on the nozzle is in the upward direction. Therefore, the anchoring force must be 482N in the downward direction.